Measurements of heat transfer from fine wires in supersonic flows

By JOHN LAUFER and ROBERT McCLELLAN
Jet Propulsion Laboratory, California Institute of Technology

(*Received* 24 *January* 1956)

SUMMARY

Results of an experimental investigation of the heat loss of fine heated wires immersed in a supersonic stream at right angles to the flow direction are presented. The measurements show that the heat loss of the wire is independent of the free-stream Mach number for values of the latter between 1·3 and 4·5. Since the wire is always in the wake of a detached shock wave, the streamlines in the neighbourhood of the wire pass through a normal shock wave. The Reynolds number Re_2, based on conditions behind a normal shock, thus becomes the characteristic parameter for the heat transfer. The measurements covering a Reynolds number range of 3 to 220 show the existence of two flow regimes. For $Re_2 > 20$ the Nusselt number is a linear function of the square root of the Reynolds number, and the equilibrium temperature is nearly independent of Re_2. For $Re_2 < 20$ the Nusselt number decreases more slowly, and the equilibrium temperature rises sharply with decreasing Reynolds number.

INTRODUCTION

The determination of the drag and heat transfer of a cylinder in a viscous compressible flow is a classical problem in fluid mechanics. At present, no theoretical solution valid for a wide range of Mach and Reynolds numbers exists. Although several accounts of experimental work on the heat transfer problem have been published (Kovásznay 1950; Lowell 1950; Spangenberg 1955; Stalder 1952; Stine 1954; Weske 1943), they disagree without exception in the manner of presenting the results; and, in cases where sufficient information is available for direct comparison, the results also disagree in absolute values. It was felt advisable, therefore, to carry out a series of experiments that might not only resolve the discrepancies found in existing measurements, but, more importantly, might throw some light on basic questions concerning (*a*) the principal parameters involved, (*b*) whether or not heat transfer from blunt bodies in compressible flows is basically different from that in low-speed flows.

With regard to (*a*), the point of view of the investigation was influenced by some recent experiments on the flow field around blunt bodies at supersonic speeds (Stine 1954; Walter 1953). These experiments have indicated that there is a tendency for the flow field behind the detached shock wave in the vicinity of the blunt body to approach a fixed pattern as the free-stream Mach number is increased. If the concept of the frozen flow field in the neighbourhood of the body is indeed correct, it would greatly simplify the heat transfer problem. It would mean that the local quantities characteristic of the frozen pattern, rather than the free-stream parameters (M_∞, Re_∞), control the heat transfer. It is therefore suggested that the local parameters should be based on conditions behind the detached shock wave (Re_2, M_2). Since, for sufficiently high free-stream Mach numbers, M_2 can be considered constant, the Reynolds number Re_2 can thus be expected to be the principal parameter of the problem. It is shown in the paper that this is in fact the case.

With regard to (*b*), at present little is known about the flow field between a blunt body and the shock wave in front of it in flows at high Mach number and low Reynolds number. Although the present experiments were not designed to investigate this problem, it was possible to obtain some information relevant to it. The measurements provide a lower Reynolds number limit ($Re_2 \sim 20$) above which the usual boundary layer approximations are expected to be valid. At lower Reynolds numbers the measurements indicate that the flow field changes, but no definite statement can be made as to the nature of this change, except that here too the parameter Re_2 correlates the results satisfactorily.

The experiments were carried out under joint sponsorship of the Department of the Army, Ordnance Corps (under Contract No. DA-04-495-Ord 18), and the Department of the Air Force. The authors wish to acknowledge valuable discussions with Drs Peter P. Wegener, Leslie Mack and H. W. Liepmann.

Symbols

- a speed of sound, cm/sec.
- c_p specific heat at constant pressure, cal gram^{-1} ° C^{-1}.
- d wire diameter, cm.
- i wire current, amps.
- h heat-transfer coefficient, cal cm^{-2} sec^{-1} ° C^{-1}.
- k thermal conductivity of air, cal cm^{-1} sec^{-1} ° C^{-1}.
- l wire length, cm.
- M u/a, Mach number.
- Nu hd/k, Nusselt number.
- p pressure, cm Hg.
- p_p Pitot pressure, cm Hg.
- Pr $c_p\mu/k$, Prandtl number.
- q heat loss from wire, cal/sec.
- R wire resistance, ohms.
- Re $\rho ud/\mu$, Reynolds number.
- T temperature, ° K.
- T_t' local stagnation temperature in boundary layer.
- t temperature, ° C.
- u velocity, cm/sec.
- x distance from flat plate leading edge, cm.
- y distance from flat plate surface, cm.
- α temperature coefficient of resistance, ° C^{-1}.
- γ ratio of specific heats.
- η $(y/x)\sqrt{(Re_\infty)}$ boundary layer parameter.
- μ absolute viscosity, poise.
- ρ density, g/cm^3.

τ temperature loading, $(T_w - T_e)/T_e$.

Subscripts

0 at 0° C.

2 conditions behind detached normal shock wave.

e equilibrium or unheated wire condition.

t stagnation condition.

w heated wire condition.

∞ undisturbed or free-stream condition.

Experimental equipment

Wind tunnel

All measurements were performed in the 20 in. supersonic wind tunnel of the Jet Propulsion Laboratory, California Institute of Technology. This tunnel is continuously operated with dry air by suitably staged centrifugal compressors, the supply pressure being variable over a range 0·2 to 4·3 atmospheres and increasing with Mach number. The supply temperature for most experiments was between 30 and 40° C. The tunnel supply temperature is known to be uniform across the wind tunnel settling chamber within 1° C. Mach numbers in small increments between 1·3 and 5·0 can be obtained in the test section with a servo-driven stainless steel flexible-plate nozzle.

Hot wires

The wire material used was an alloy of 90% platinum and 10% rhodium. The nominal wire diameters were 0·00127 cm (0·0005 in.) and 0·00038 cm (0·00015 in.) as specified by the manufacturer. No attempt was made to determine wire diameters independently since use of the nominal values yielded consistent data in the final results. The length to diameter ratio, or aspect ratio, of the wires was approximately 400 for the larger diameter wires and 550 for the smaller ones. The wire holder shown in figure 1 (plate 1) was made of two stainless steel wedge-shaped prongs insulated electrically from one another by a layer of glass cloth bonded to the metal with a Teflon bonding agent. The wires were attached to the prongs of the holder with soft solder.

Heating circuit

The direct current for heating the wires was supplied by a 24-volt storage battery through a circuit including a Wheatstone bridge for the measurement of the wire resistance. A Brown null indicator showed the bridge balance. A Rubicon portable precision potentiometer was used to measure the potential drop across a precision resistor of 1 ohm in series with the hot wire.

Pitot probe

In order to determine the local Mach number accurately, a Pitot probe was placed close to the hot wire. The Pitot pressure was measured on a mercury micromanometer to an accuracy of $\pm 0{\cdot}1$ mm mercury.

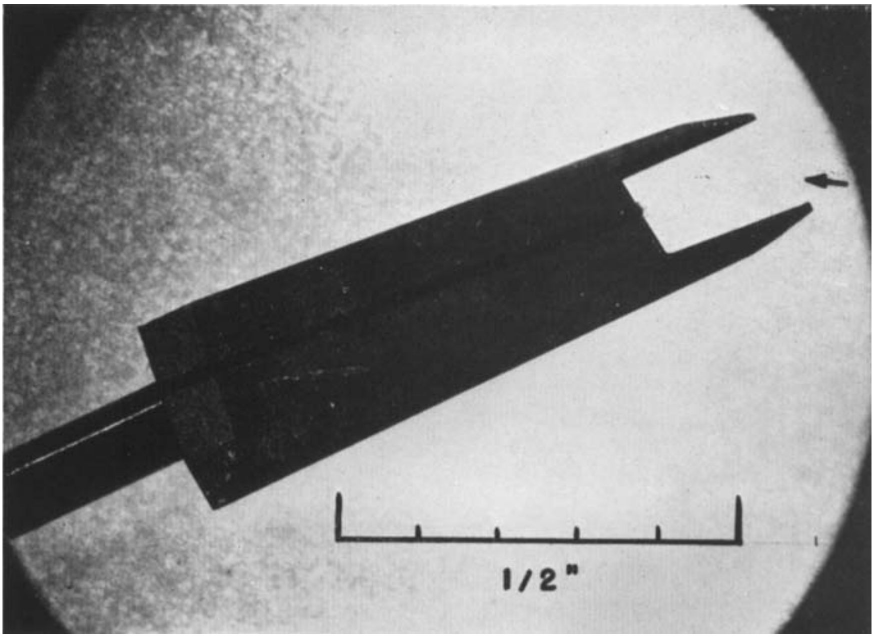

Figure 1. Hot-wire probe.

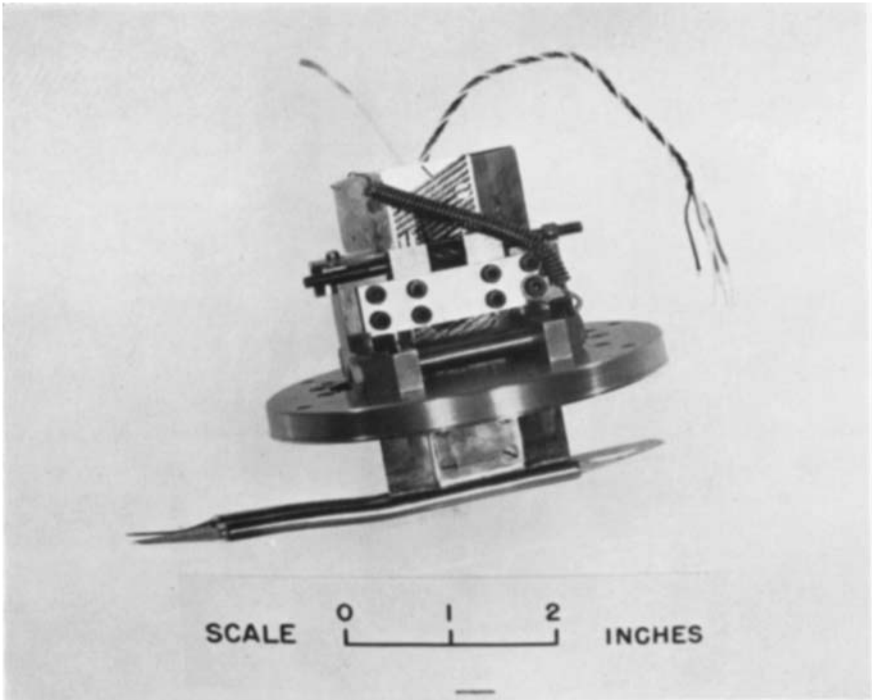

Figure 2. Traversing mechanism.

Traversing mechanism

The boundary layer survey presented was made on a smooth flat plate which spanned the working section of the tunnel. A traversing mechanism (figure 2, plate 1) fitted in the plate could be positioned to an accuracy of $\pm 0{\cdot}01$ mm.

EXPERIMENTAL PROCEDURE

For the determination of the local flow conditions, the standard methods of recording the tunnel stagnation pressure, stagnation temperature, and the local Pitot pressure were used. The accuracy of the measured Mach numbers is believed to be within $\frac{1}{2}\%$, and the values of the Reynolds numbers are accurate within 2%.

In order to obtain the heat loss coefficient, or Nusselt number, a measurement of wire resistance and current is required, together with a knowledge of the heated and unheated wire temperatures. Since measurements of resistance and current can be carried out with great precision by well-known methods, only the accurate establishment of the wire temperatures necessitated special effort. This involved the measurement of the thermal coefficient of resistance α, carried out in the following manner. The resistance of several samples of the 0·00127 cm and 0·00038 cm diameter wires was measured at 0° C, at room temperature, and near the temperature of boiling water. The coefficient α, based on 0° C, was computed from $R = R_0(1+\alpha t)$ with R being the resistance of the wire at some temperature t° C. It was considered that within the temperature range 0° C to 300° C covered in the present experiment, the use of a linear resistance–temperature relationship was acceptable. A typical calibration curve is shown in figure 3. It was found that for the large diameter wire $\alpha = 0{\cdot}00175/^\circ\,\mathrm{C} \pm \frac{1}{2}\%$, and for the smaller wires $\alpha = 0{\cdot}00166/^\circ\,\mathrm{C} \pm 1\frac{1}{2}\%$. The percentages refer to the maximum variations of the thermal coefficient of resistance for various samples taken from the same spool of wire. The above values of α were used in reducing all the heat loss data except at Mach numbers 1·33 and 4·54. In these cases a wire from a different spool with $\alpha = 0{\cdot}00169/^\circ\,\mathrm{C}$ was used.

In principle, having the information on α and R_0 and a resistance measurement for a given flow condition, the wire temperature could be measured. Unfortunately, it was found that during several measurements the wire stretched permanently, presumably due to the wind tunnel starting loads, or even due to continuous high air loads. The wire stretching increased the resistance by several percent. This fact was established by a resistance–temperature recalibration of a few wires after several hours of exposure to the air stream. This occasional stretching made it impossible to accurately infer the wire temperature from its measured resistance. It was therefore decided to make a number of careful separate measurements of the equilibrium (unheated) resistance of wires exposed to the free stream in the entire Mach number and Reynolds number range, and for this purpose some specially constructed wires were used. The length of these wires was somewhat smaller than that used for the later heat loss measurements in order to increase the probability of test survival. (Calculations indicated that in the case of unheated wires, the correction due to heat loss through the prongs of the holder was within the experimental scatter.) Furthermore, the resistance of all the wires used in this set of experiments was calibrated against temperature before and after the measurements. If the resistance had changed more than a few tenths of one percent due to exposure to the air stream, the results were discarded. With this technique it was possible to obtain wire equilibrium measurements repeatable within $\pm 1\%$.

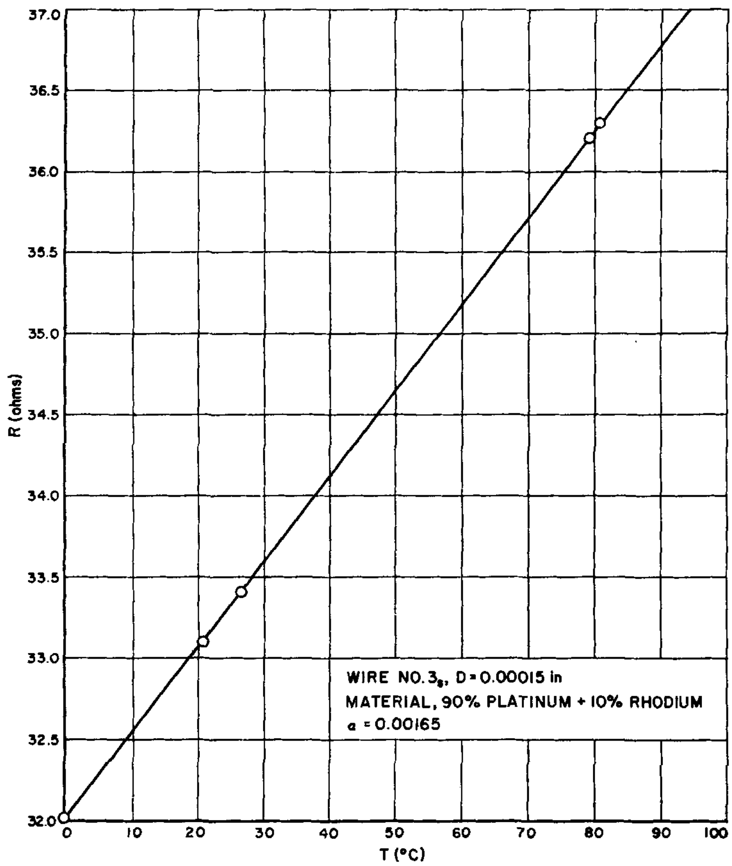

Figure 3. Temperature–resistance calibration.

The procedure for the heat loss measurements may be described as follows. With the wire in the free stream, its 'cold' resistance was measured, that is, the resistance of the wire when there is no heating current in it, as in measurements of the wire recovery temperature. Next the wire was heated by passing a current through it until its resistance reached some predetermined value, and this current was measured. The sequence was then repeated for a different predetermined value of resistance. Simultaneously with these measurements, the tunnel supply pressure and temperature were recorded, and the Pitot pressure in the vicinity of the wire was measured. The entire procedure was repeated at several tunnel pressure levels at a given Mach number, so as to give as wide a variation of Reynolds number as possible. In turn the Mach number was also varied over the entire range of the tunnel. Figure 4 shows a typical set of these measurements at $M_\infty = 3{\cdot}05$.

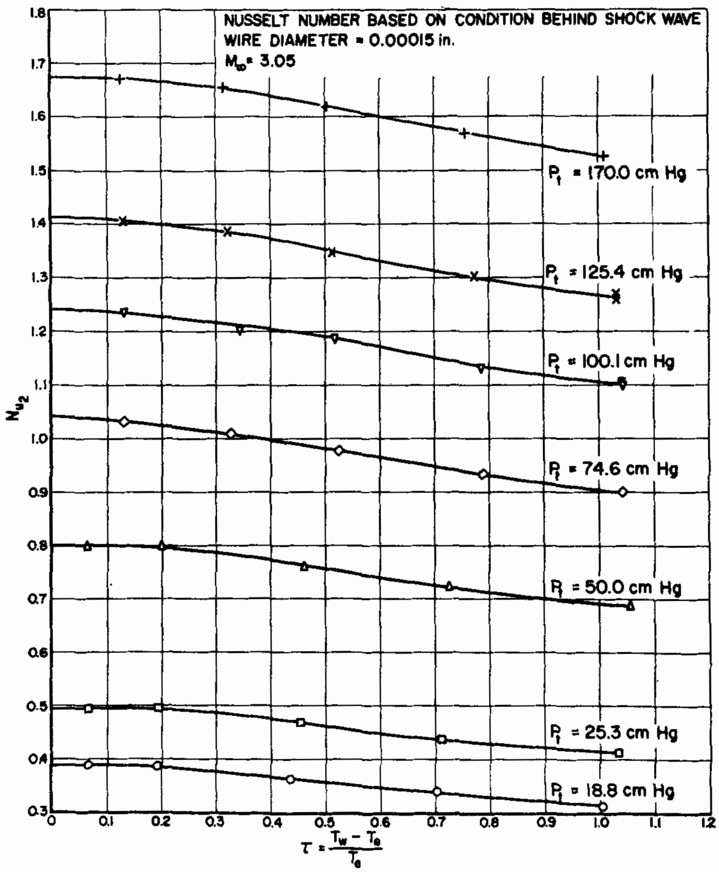

Figure 4. Typical measurements at $M_\infty = 3{\cdot}05$.

Data reduction

The measurement of the wire heat loss involved the recording of the following quantities.

(*a*) *The flow parameters:* stagnation pressure p_t, Pitot pressure p_p, and stagnation temperature T_t. From these the local Mach and Reynolds numbers were obtained. The flow conditions on which the Reynolds number was based will be discussed later.

(*b*) *The wire parameters:* current i, unheated or equilibrium resistance R_e, heated wire resistance R_w. From the values of R_e measured at various Mach and Reynolds numbers, the equilibrium wire temperatures could be computed using the relation

$$T_e = \frac{R_e - R_0}{R_0 \alpha} + 273^\circ \text{ C}, \tag{1}$$

where α and R_0 were obtained from the resistance–temperature calibrations. Thus the functional relationship

$$\frac{T_e}{T_t} = f(M_\infty, Re_\infty) \tag{2}$$

could be established.

Only wires that survived a complete run and showed no change in R_0 on recalibration were used in the determination of this function. Once established, this relationship permitted the computation of T_e from the known values of T_t, M_∞ and Re_∞ in subsequent measurements.

The heat loss from the wire was expressed in the form of a Nusselt number

$$Nu = \frac{hd}{k},$$

where

$$h = \frac{q}{\pi l d(T_w - T_e)}$$

is the heat transfer coefficient for a cylinder. In terms of electrical quantities,

$$Nu = \frac{q}{\pi l k(T_w - T_e)} = \frac{0{\cdot}239 i^2 R_w R_e}{\pi l k(R_w - R_e)[1+\alpha(T_e - 273)]},$$

where 0·239 is a conversion factor from cal/sec to watts. The temperatures on which the values of the heat conductivity of air were based will be described later.

Since R_e was directly measured and T_e was obtained from the previously established experimental relation (2), any change in R_0 due to stretching of the wire during the supersonic measurement could be immediately detected, and the correct value of R_0 determined. This change was always less than 3% of the initial value of R_0. In this simple correction it was assumed that changes in resistance resulted from uniform changes in the length and diameter of the wire.

Due to the fact that the hot wire is of finite length and the holder is at a lower temperature than the wire itself, there is heat conduction to the holder as well as heat loss to the air stream by forced convection. This means that the measured heat loss per unit length for a wire of finite length is different than it would be for a wire of infinite length in the same air stream with the same heating current. The correction for this end loss effect was given first by King (1914). The same method was used here adopting the technique of computation of Kovásznay (1950). The corrections were of the order of 5% of the measured Nusselt number.

Results and discussion

The principal aim of the present investigation was, first, to find the important flow parameters that govern the heat loss from a heated wire in supersonic flow, and, second, to obtain an empirical relationship between these flow parameters and the measured heat loss.

In general it can be shown from dimensional arguments that the heat loss, usually expressed in terms of a nondimensional Nusselt number, depends on the following quantities (see Kovásznay 1950):

$$Nu_\infty = Nu(Re_\infty, M_\infty, Pr_\infty, \gamma, \tau, l/d). \tag{3}$$

All the parameters are referred to free-stream conditions, which are usually given in the problem. The Prandtl number and the ratio of the specific heats of the gas were very nearly constant in the present experiments. The aspect ratio l/d enters into the problem mainly because of conduction losses through the wire supports. A suitable small correction described earlier was made to eliminate this effect. The above functional relationship simplifies, therefore, to

$$Nu_\infty = Nu(Re_\infty, M_\infty, \tau). \tag{4}$$

The heat transfer measurements presented in this form exhibited a systematic variation with free-stream Mach number. The question arose whether, with a more judicious choice of parameters, further simplification could be attained.

In this connection the experiments of Walter & Lange (1953) are of interest. They measured the local recovery temperature and the pressure distribution around an insulated cylinder in supersonic flow. Their measurements indicated that the flow conditions behind the detached shock wave and in the vicinity of the cylinder were very nearly independent of the free-stream Mach number in the range investigated ($2{\cdot}5 < M < 5{\cdot}0$). Although the Reynolds numbers of these experiments were much higher than those met here, the cylinder boundary layer was laminar, and their conclusions can be expected to hold also in the lower Reynolds number range. This immediately suggests that the parameters in (4) should be expressed in terms of the conditions beyond the detached normal shock wave, rather than the undisturbed flow conditions. Accordingly, (4) can be rewritten

$$Nu_2 \frac{k_2}{k_\infty} = Nu\left(Re_2 \frac{\mu_2}{\mu_\infty}, M_2, \tau\right). \tag{5}$$

The ratios of heat conductivity and viscosity depend very nearly on the temperature ratio T_2/T_∞, which in turn is a function of M_2 only. Therefore (5) becomes

$$Nu_2 = Nu(Re_2, M_2, \tau).$$

As the supersonic free-stream Mach number increases, M_2 approaches a constant. Thus,

$$Nu_2 = Nu(Re_2, \tau) \qquad \text{for } M_\infty \gg 1. \tag{6}$$

All the measurements were expressed in terms of these parameters and are discussed below.

Figure 5 shows the results of measurements of the equilibrium temperature attained by the unheated wire at various Mach and Reynolds numbers. It is seen that for a Reynolds number Re_2, larger than approximately 20, the equilibrium temperature variation with Re_2 and M is negligible, being within the limits of the experimental accuracy ($\pm 1\%$). Unfortunately, the Reynolds number range is not very wide at the higher Mach numbers; but the results of Walter & Lange (1953) strongly suggest that the equilibrium temperature has a constant value. Below a Reynolds number of 20, on the other hand, a sharp rise in the equilibrium temperature was observed, and values above the stagnation temperature were measured.

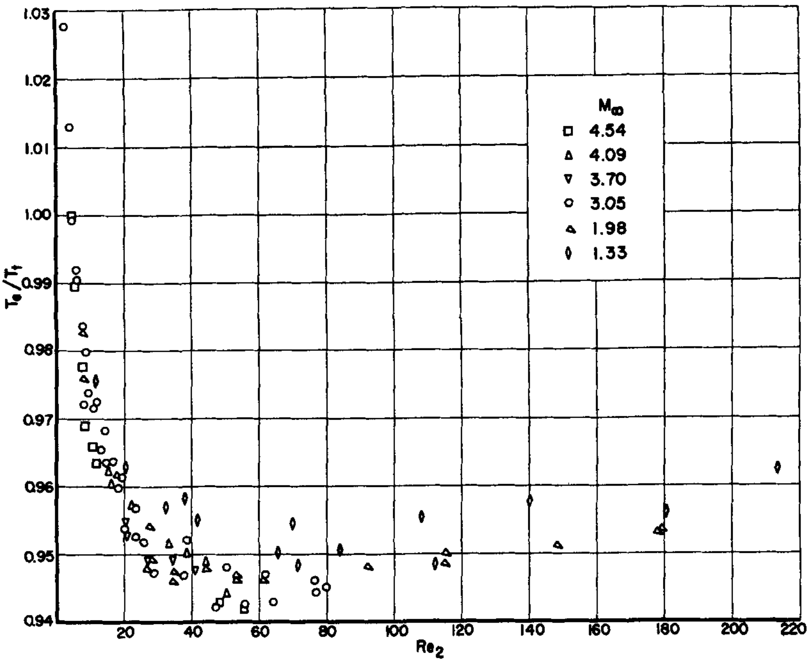

Figure 5. Variation of equilibrium temperature.

The results of the heat loss measurements are shown in figure 6. The Nusselt number is plotted as a function of the Reynolds number with the temperature loading as a parameter. Both Nusselt and Reynolds numbers are expressed in terms of conditions behind the normal detached shock wave. The points on the figure were directly read from plots of the type shown in figure 4. It is seen that the measurements presented in this manner do not exhibit a dependence on the free-stream Mach number in accordance with equation (6), even for a Mach number as low as 1·3. Further, provided $Re_2 > 20$, the Nusselt number varies with the square root of the Reynolds number—a relationship well-known for low speed flows. Again, below a Reynolds number of about 20 ($\sqrt{(Re_2)} \sim 4{\cdot}5$) the square root relationship breaks down, and the heat loss decreases at a slower rate with decreasing Reynolds number.

From these results it is apparent that the predominant parameter of the problem is the Reynolds number. They also substantiate the conjecture made earlier that the Reynolds number should not be based on the undisturbed free-stream conditions, and that the 'apparent free stream' of the cylinder was the flow behind the detached shock wave. For convenience, and because in front of the body the shock wave was in fact very nearly normal, the Reynolds number was based on conditions behind a normal shock wave.

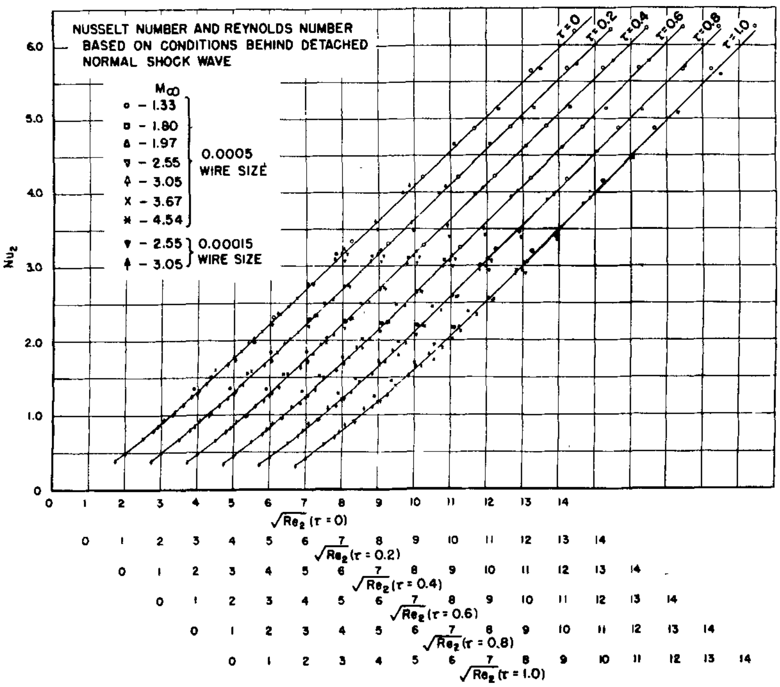

Figure 6. Nusselt number variation at various temperature loadings.

The measurements can evidently be discussed in terms of two Reynolds number ranges, a Reynolds number of approximately 20 being the dividing line.

High Reynolds number flows: $Re_2 > 20$

As pointed out earlier, in this range the heat loss coefficient varied with the square root of the Reynolds number for a given temperature loading. Such a relationship was found to exist in low-speed subsonic and transonic streams (King 1914; Lowell 1950; Stalder 1952), and is characteristic of flows where boundary layer approximations are applicable. The results suggest, therefore, that above $Re \sim 20$ the usual boundary layer approximation could be used in the theoretical treatment of this problem.

Spangenberg's data (1955) for $M_\infty = 1{\cdot}25$ to $M_\infty = 1{\cdot}9$ are found to agree with the present results within the scatter of his experiments.

Kovásznay and Törmarck (1950) were the first to point out the independence of the wire heat loss and the free-stream Mach number, but their values are too high by about 20 to 25%.

Low Reynolds number flows: $Re_2 < 20$

In this range the measurements indicate that (*a*) the equilibrium temperature of the wire increases rapidly with decreasing Reynolds number (figure 5), and exceeds the stagnation temperature, and (*b*) the heat loss decreases at a slower rate with decreasing Reynolds number (figure 6). The former effect was first noticed by Stalder, Goodwin & Creager (1952). They explained it from the point of view of kinetic theory, and related the equilibrium temperature to the ratio of the mean free path λ and cylinder diameter. It is interesting to note that the Reynolds number Re_2 is equivalent to d/λ, the reciprocal of the parameter used by Stalder, Goodwin & Creager, provided the viscosity relation of kinetic theory is used and the assumption $M_\infty \geqslant 1$ is made.

At this point one may inquire if, in the region where the mean free path and cylinder diameter are of the same order of magnitude, conditions can still be adequately described from the continuum mechanics point of view, and if it is in fact necessary to formulate the problem in terms of the kinetic theory. At present there is not enough evidence for a definite answer. However, from the fact that in the present experiments one Reynolds number alone—without additional parameters—correlates the measurements in this range as well as in the high Reynolds number range, one may expect continuum flow theories to be valid for the flow regime in question.

Conclusion

Measurements of the heat loss of fine cylinders in supersonic flows within a Mach number range 1·3 to 4·5 indicate the following results:

(1) The principal independent parameter of the problem is the Reynolds number based on conditions behind the detached normal shock wave.

(2) For $Re_2 > 20$, the cylinder equilibrium temperature is independent of both Reynolds number and free-stream Mach number and has a value $T_e/T_t = 0{\cdot}95 \pm 0{\cdot}01$; the Nusselt number Nu_2 varies with the square root of the Reynolds number for a given temperature loading and is independent of the free-stream Mach number.

(3) For $Re_2 < 20$, the equilibrium temperature rises sharply with decreasing Reynolds number; in fact it exceeds the stagnation temperature at about $Re_2 < 5$. The square root relationship between Nusselt and Reynolds numbers does not hold; in this region also, the free-stream Mach number is not a parameter of the problem.

Appendix

Mean flow measurements in a supersonic laminar boundary layer by means of a hot wire

The results described in this paper suggest the use of a hot wire for measurements of some mean flow quantities in an unknown supersonic flow field. As an example, hot-wire measurements were made in a laminar boundary layer on a flat plate 24 cm from the leading edge at a stagnation pressure of 24·0 cm Hg and a stagnation temperature of 285° K. The free-stream Mach number was 3·05. The measurements consisted of recording the unheated wire resistance and wire heat loss for a constant temperature loading at different distances from the flat surface. It is shown that by these two types of measurements and by the use of the results of this paper in a modified form, the mass flow ρu and local stagnation temperature T_t' can be obtained. Furthermore, the assumption of constant static pressure across the boundary layer allows the calculation of all the other flow parameters.

The results were expressed in a more convenient form the following way. It was shown that, for a given temperature loading,

$$Nu_2 = Nu(Re_2).$$

The fluid properties, heat conductivity and absolute viscosity are based on conditions behind the normal detached shock wave which are not known *a priori*. The above relationship can be written

$$Nu_w \frac{k_w}{k_e} \frac{k_e}{k_2} = f\left(Re_w \frac{\mu_w}{\mu_e} \frac{\mu_e}{\mu_2}\right).$$

Since both the heat conductivity and viscosity are functions of the temperature only, the ratios k_w/k_e and μ_w/μ_e depend very closely on T_w/T_e, which is a constant for a given temperature loading. Moreover, it is found that the ratio T_e/T_2 is very nearly unity within the Mach and Reynolds number range of the experiments. The law of heat loss from the hot wire can therefore be written in the form

$$Nu_w = Nu(Re_w, \tau).$$

Figure 7 shows the measurements evaluated in this form. The points on this figure were obtained by cross-plotting from curves of Nu_w versus τ for constant values of Re_w, similar to plots shown in figure 4. It might be mentioned that the heat loss law in a form

$$Nu_e = Nu(Re_e, \tau)$$

could also have been used.

Furthermore, the variation of equilibrium temperature with Reynolds number was put in the form

$$\frac{T_e}{T_t} = f(Re_e).$$

The method of data reduction may be described as follows.

(1) From the measurement of Re the equilibrium temperature is calculated by means of the relation

$$T_e = \frac{R_e - R_0}{\alpha R_0} + 271^\circ \text{ C.}$$

(2) From the heat loss measurements Nu_w is computed; and, using figure 7, the mass flow ρu can be obtained, since wire diameter and the viscosity based on wire temperature are known.

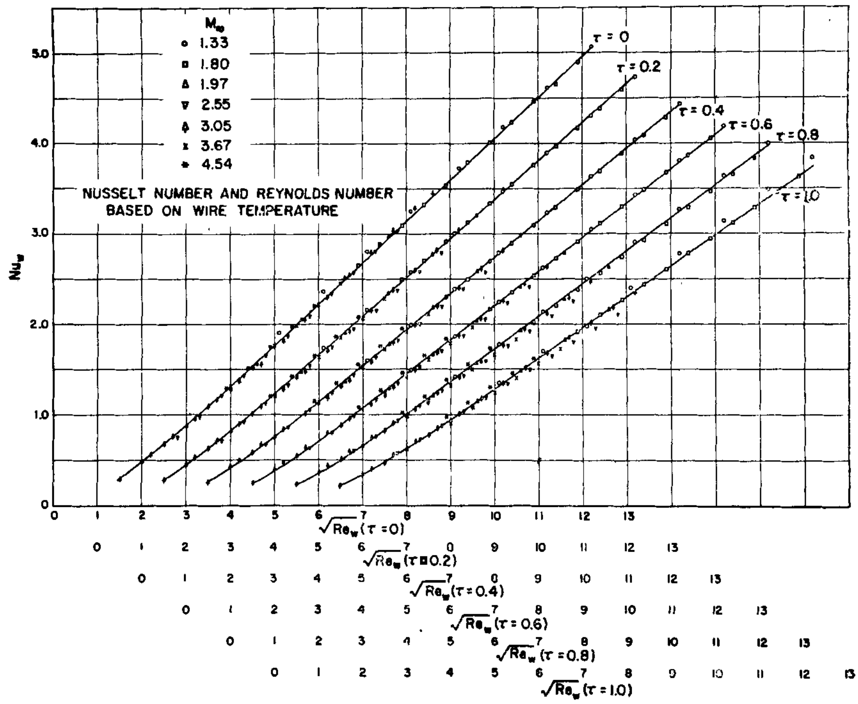

Figure 7. Nusselt number variation at various temperature loadings.

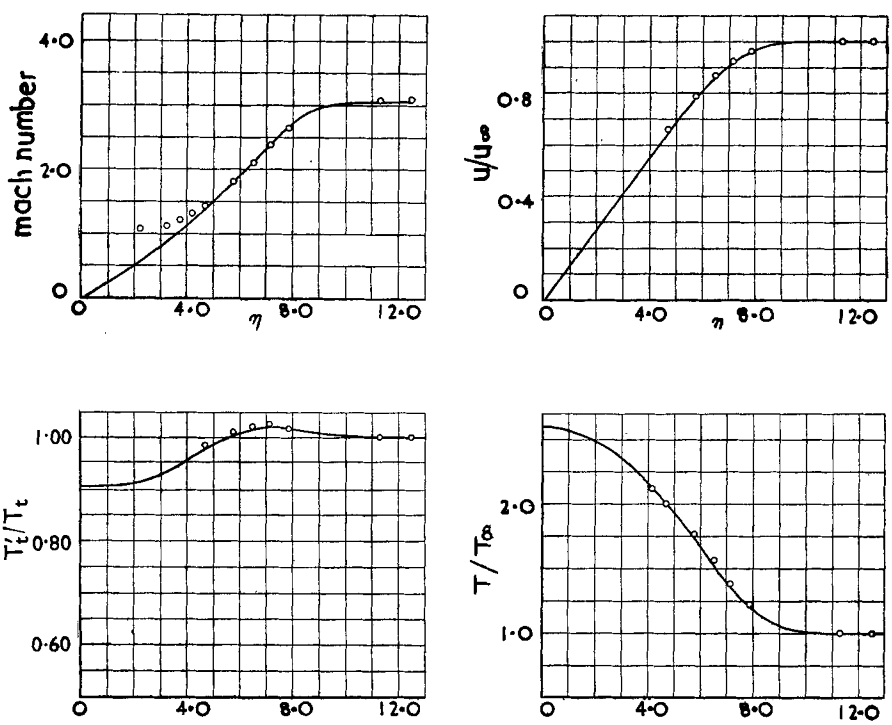

Figure 8. Distributions of mean flow parameters in a laminar boundary layer at $M_\infty = 3{\cdot}05$ $\left(\eta = \frac{y}{x}\sqrt{Re_\infty}\right)$.

(3) From the relation
$$\frac{T_c}{T_t} = f(Re_e)$$
the local total temperature is found.

(4) Finally, by assuming constant static pressure across the boundary layer, the usual determination of Mach number, mean velocity and density can be made.

Figure 8 shows the final results. The solid lines are theoretical values obtained by using the method of Klunker & McLean (1953).* From the Mach number distribution it is seen that the measured points below a Mach number 1·3 deviate significantly from the theoretical values. Since the heat loss variation shown in figure 7 is not expected to hold in this range, the discrepancy is not surprising. Therefore, results obtained near the wall where $M < 1\cdot3$ are not plotted in the other distributions shown on this figure. The figure indicates a very good agreement between theory and experiment in the region considered ($M > 1\cdot3$). It is believed that the useful range of the hot-wire method employed here could be extended to lower Mach numbers using the results of Spangenberg (1955) and by devising a more elaborate technique.

References

King, L. V. 1914 On the convection of heat from small cylinders in a stream of fluid: determination of the convection constants of small platinum wires with applications to hot-wire anemometry, *Phil. Trans.* A, **214,** 373.

Klunker, E. B. & McLean, F. E. 1953 Effect of thermal properties on laminar-boundary-layer characteristics, *Nat. Adv. Comm. Aero., Wash., Tech. Note* no. 2916.

Kovasznay, L. S. G. & Törmarck, S. I. A. 1950 Heat loss of hot wires in supersonic flow, *Bumblebee Rep.* 127, *Aero. Dept., Johns Hopkins University.*

Lowell, H. H. 1950 Design and applications of hot-wire anemometers for steady-state measurements at transonic and supersonic airspeeds, *Nat. Adv. Comm. Aero., Wash., Tech. Note* no. 2117.

Spangenberg, W. G. 1955 Heat-loss characteristics of hot-wire anemometers at various densities in transonic and supersonic flow, *Nat. Adv. Comm. Aero., Wash., Tech. Note* no. 3381.

Stalder, J. R., Goodwin, G. & Creager, M. O. 1952 Heat transfer to bodies in a high-speed rarefied-gas stream, *Nat. Adv. Comm. Aero., Wash., Rep.* no. 1093.

Stine, H. A. 1954 Investigations of heat transfer from hot wires in the transonic speed range, *Proc.* 1954 *Heat Transfer and Fluid Mechanics Institute.*

Stine, H. A. & Wanlass, K. 1954 Theoretical and experimental investigation of aerodynamic heating and isothermal heat-transfer parameters on a hemispherical nose with laminar boundary layer at supersonic Mach numbers, *Nat. Adv. Comm. Aero., Wash., Tech. Note* no. 3344.

Walter, L. W. & Lange, A. H. 1953 Surface temperature and pressure distribution on a circular cylinder in supersonic cross-flow, *NAVORD Rep.* 2854.

Weske, J. R. 1943 Methods of measurement of high air velocities by the hot-wire method, *Nat. Adv. Comm. Aero., Wash., Tech. Note* no. 880.

* The computation was carried out on an electronic digital computing machine by Dr Leslie Mack in connection with some laminar boundary layer stability calculations. The authors are indebted to him for providing them with this information.